LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **APRIL 2019**

16/17UMT3MC02– VECTOR ANALYSIS AND ORDINARY DIFF. EQUATIONS

Date: 25-04-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

Answer ALL questions **(10 X 2 = 20 Marks)**

1. Find the unit vector normal to the surface $x^2 + 3y^2 + 2z^2 = 6$ at the point (2, 0, 1).
2. Find the value of 'a' so that the vector $\vec{F} = (z + 3y)\vec{i} + (x - 2z)\vec{j} + (x + az)\vec{k}$ is solenoidal.
3. Evaluate $\int_C \vec{F}.d\vec{r}$ where $\vec{F} = x^2\vec{i} + y^2\vec{j}$ along the line $y = x$ from $A(0, 0)$ to $B(1, 1)$.
4. Show that the value of the integral $\int_C \vec{F}.d\vec{r}$ is independent of the path C, where $\vec{F} = (e^x z - 2xy)\vec{i} + (1 - x^2)\vec{j} + (e^x + z)\vec{k}$.
5. State Stoke's theorem.
6. Show that $\iint_S \vec{r}.\hat{n}\, dS = 3V,$ where V is the volume enclosed by a closed surface S.
7. Solve $\frac{dy}{dx} - \frac{y+2}{x-1} = 0$.
8. Solve $p^2 - 5p + 6 = 0$.
9. Find the complete integral of $(D^2 + 16)y = 2$.
10. Convert $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \sin(\log x^2)$ into linear differential equation with constant coefficients.

PART – B

Answer Any FIVE Questions **(5 X 8 = 40 Marks)**

11. Prove that $div\,\vec{r} = 3$ and $curl\,\vec{r} = 0$, where $\vec{r}$ is the position vector at the point (x, y, z).
12. If $u = x + y + z,\ v = x^2 + y^2 + z^2,\ \ w = yz + zx + xy,$ Prove that $(\nabla u).(\nabla v \times \nabla w) = 0$.
13. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $\overline{|r|} = r$, then prove that $\nabla(r^n) = nr^{n-2} \cdot \vec{r}$.
14. Evaluate $\int_C xydx + xy^2dy$ by Stoke's theorem where C is the square in the XY plane with vertices (1, 0) , (-1, 0), (0, 1) and (0, -1).
15. Solve $\frac{dy}{dx} + y\cos x = \frac{1}{2}\sin 2x$.
16. Solve $xp^2 - yp - x = 0$.
17. Solve $(D^2 - 2D + 1)y = e^{3x}$.
18. Solve $(D^2 + 3D + 2)y = \sin x$.

PART – C

Answer Any TWO Questions **(2 X 20 = 40 Marks)**

19. (i) A field $\vec{F}$ is of the form $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$. Show that $\vec{F}$ is conservative field. **(10)**

(ii) Prove that $\nabla . [\nabla r^n] = n(n+1)r^{n-2}$. **(10)**

20. (i)Evaluate $\iint_S \vec{F}.\hat{n}ds$ where $\vec{F} = z\,\vec{i} + x\,\vec{j} - 3y^2 z\vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. **(10)**

(ii) Using Green's theorem, evaluate $\oint_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$ where C is the closed region bounded by $x = 0, y = 0, x + y = 1$. **(10)**

21. Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelopiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$. **(20)**

22. Solve the equation $\frac{d^2y}{dx^2} + a^2 y = \sec ax$ by the method of variation of parameters. **(20)**

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